**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans.** Since work beings 10 mins after the car is dropped, the time left to complete work is 50 min.

Probability that Service Manager cannot meet his commitment = P(X>50) = 1 – Pr (X <= 50) (X is the time taken to complete work). Convert 50 to z-score

Standard normal variable Z = (X-*μ*)/*σ* = (X-45)/8

P (X <= 50) = P (Z <= (50-45)/8) = PR (Z <= 0.625) = 0.73237 = 73.237% (the number in z- table)

Probability that service manager will not meet his commitment is: 100 – 73.237 = 26.736 % = 0.2676

So, the answer is **B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans.**

*μ* = 38 & *σ* =6

1. Probability of employees > 44= Pr (x > 44) =1-Pr(x<=44)

Z=(X-*μ*)/*σ* = (x-38)/6

Pr(x<=44) = Pr(z<=(44-38)/6) = Pr(Z-1) = 0.84134 = 84.134%

Probability that employees will be greater that 44 = 100-84.134 = 15.866

Probability of employees between 38 & 44 = Pr (x<=44) – Pr (x>=38)

Here, Pr(x< = 44) = 0.84134

Pr (x>=38) = Pr (z>=(38-38)/6) = Pr (z>=0)=0.5

Therefore, Pr (x<=44) – Pr (x>=38) = 0.84134 - 0.5 = 0.34134 = 34.134%

So, the statement “More employees at the processing center are older than 44 than between 38 and 44” is **TRUE**

1. Probability of employees less than 30 = Pr (x<30)

Z=(X-*μ*)/*σ* = (30-38)/6

Pr (x<30) = Pr (z<(30-38)/6) = Pr (z< -1.3333) = 0.09176 = 9.176 %

So, the number of employees with probability 0.0917 of them being under

30=400\*0.0917 = 36.68 = 36

The statement of “training program for employees under the age of 30 at the center would be expected to attract about 36 employees” is **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans.** According to the Central Limit Theorem, any large sum of independent, identically distributed (iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean μ, and the variance, σ2 and written as X ~ N (μ, σ2).

Given X1 ~ N (μ, σ2) and X2 ~ N (μ, σ2) are two independent identically distributed random variables.

From the properties of normal random variables,

if X ~ N (μ1, σ12) and Y ~ N (μ2, σ22)  are two independent identically distributed random variables then

the sum of normal random variables is given by:

X + Y ~ N (μ1+ μ2, σ12+ σ22)

and the difference of normal random variables is given by

X - Y ~ N (μ1- μ2, σ12+ σ22)

When  https://tex.z-dn.net/?f=Z%20%3D%20aX, the product of X is given by

Z ~ N (aμ1, a2σ12)

When  https://tex.z-dn.net/?f=Z%20%3D%20aX%20%2B%20bY, the linear combination of X and Y is given by

Z ~ N (aμ1+ bμ2, a2σ12+ b2σ22)

Given to find, 2X1

Thus, following the property of multiplication,

we get 2X1~ N (2μ, 22 σ2) ⇒ 2X1~ N (2μ, 4σ2)

and following the property of addition,

X1 + X2 ~ N (μ + μ, σ2 + σ2) ~ N (2μ + 2σ2)

And the difference between the two is given by

2X1 – (X1 + X2) ~ N (2μ - 2μ, 2σ12 + 4σ22) ~ N (0,6σ2)

The mean of 2X1 and X1 + X2 is same but the var(σ2) of  2X1 is 2 times more than the variance of X1 + X2 .

The difference between the two says that the two given variables are identically and independently distributed.

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans.** Given:  p(a<x<b) = 0.99 , Mean =100, Standard Deviation = 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details, we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python:

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

              Z (0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

              Z (99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = - (20\*2.576) + 100= 48.5

      b = (20\*2.576) + 100= 151.5

Two values symmetric about mean for the given standard normal distribution are [48.5, 151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.** Given that:

$1 = Rs. 45

Profit1 ~ N (5, 32)

Profit2 ~ N (7, 42)

Thus,

Company's profit:

P ~ N (5 + 7, 32 + 42) = N (12, 52)

A):

95% of the probability lies between 1.96 standard deviations of the mean.

Thus range is:

= (12 – 1.96 \* 5, 12 + 1.96 \* 5)

= ($2.2 M, $22.8M)

= (Rs. 99M, Rs. 1026M)

B):

Fifth percentile is calculated as:

P [Z ≤ (p - 12) / 5] = 0.05

From p values of z score table, we get:

(p - 12) / 5 = -1.644

P = 12 – 8.22 = 3.78

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.

Or 5th percentile of profit is Rs. 170.1 Million.

C):

Loss is when profit < 0

Thus: p < 0

The first division of company, thus have larger probability of making a loss in a given year.